THE COST OF COLLEGE EDUCATION PROJECT PACKET
Teacher Guide

Introduction

We live in a society where a college education is considered the norm and not the exception. While everyone is expected to attend a college or university (of their choice), most American families don't have a plan for paying for this expectation. According to Sallie Mae's "How America Saves for College 2013" study, just 36 percent of middle-income families and 29 percent of low-income families have set aside money to pay for furthering their education.

As Grandma, Helen, watched her proud son hold her beautiful new granddaughter Jessica, she thought, "What can I do help her live the wonderful life she so richly deserves?" The grandmother reflected on the times she and her husband struggled to keep their son in school and what a financial burden the student loans had become for her son. Because her son saw no way out of this financial maze the opportunity for a family seemed almost nonexistent. She realized that there was a real possibility she could have never experienced this type of joy. She quickly realized that this was not the fate she wanted her granddaughter to suffer. She knew she had to do something to not have her granddaughter suffer the same fate.

Mathematical Goals

- Represent data on a scatter plot
- Describe how two variables are related
- Use technology to graph and analyze functions
- Informally assess the fit of a function by plotting and analyzing residuals

Essential Question

How can students predict and prepare for the cost of a college education?

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

Part 1: Future Value

The Grandmother, Helen, is looking for a plan to finance her new grandchild Jessica's college education. She is planning to invest $50,000 on Jessica's first birthday, July 1, 2008. Your role is to assist the Grandma in finding an investment plan which has to offer compound interest for the investment. In addition, you need to predict Jessica's future college education cost by using the available data. Use the internet or any reliable sources, find the interest rate and the number of compounding periods per year advertised by a bank.

- The principal (P) is $50,000.
- Research the annual interest rate (r) for the investment. r = *answer may vary*
- State the time (t) in years for the investment (as in when Jessica will be attending her first year of college). t = *17*
- State the number of compounding periods (n) per year. n = *answer may vary*

1. Write a mathematical model for the future value of Grandma's investment as an exponential function, with time (t) as the independent variable. *Use $F(t) = P(1 + {}^r/_n)^{nt}$*

    Sample solution:
    $P = 50000$ $r = 4.2\%$ $n = 4$
    $F(t) = 50000\left(1 + \frac{0.042}{4}\right)^{4t}$
    $F(t) = 50000(1.0105)^{4t}$

2. Write the growth factor for this investment. Round your answer up to five decimal places. *(Note: growth factor = 1 + r/n)*

    Sample solution:
    Growth factor $= 1 + \frac{0.042}{4}$ $= 1.01050$

3. Calculate the future value of Grandma's investment on Jessica's 17th birthday. Round your answer to the nearest dollar.

    Sample solution: On the 17th birthday, t = 16
    $F(16) = 50000(1.0105)^{4(16)}$
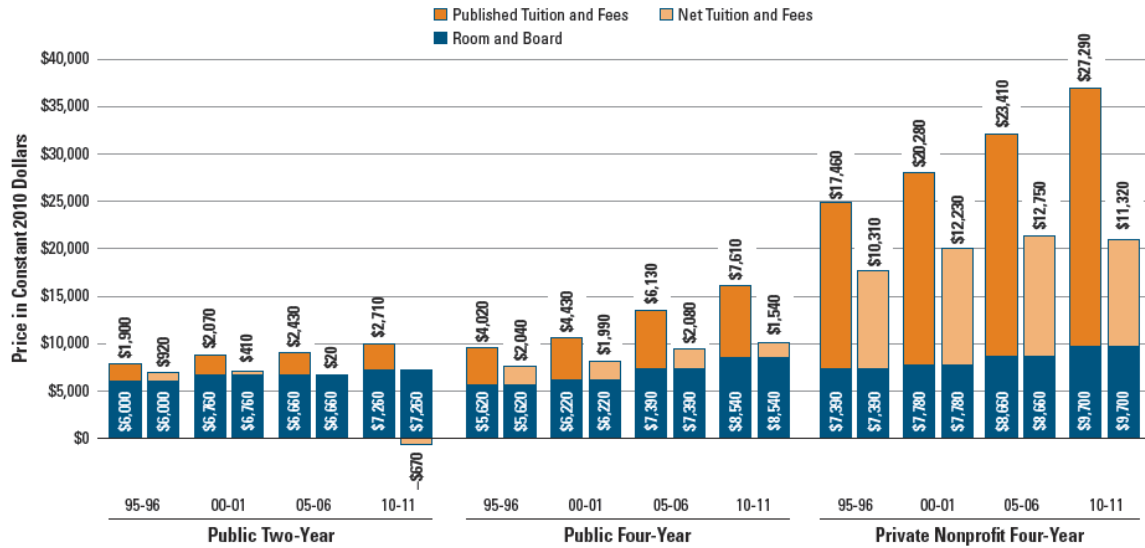   * $= 50000(1.0105)^{64}$*
    * $= 97565.07$*
    The future value of Grandma's investment on Jessica's 17th birthday is $97565

Part 2: College Tuition and Fee Trend Table

FIGURE 7 Published Tuition and Fees, Net Tuition and Fees, and Room and Board in Constant 2010 Dollars, Full-Time Undergraduate Students, 1995-96, 2000-01, 2005-06, and 2010-11 (Estimated)

The distribution of grant aid across sectors and between full-time and part-time students is based on data from the National Postsecondary Student Aid Study. The distribution for 2009-10 includes estimates of the changes resulting from the large increase in Pell Grants that year. Because financial aid data for 2010-11 are not yet available, net prices for 2010-11 are estimated based on past years and available information about changes in financial aid.

NOTE: Numbers have been rounded to the nearest $10.

SOURCES: The College Board, *Annual Survey of Colleges*; *Trends in Student Aid 2010*; calculations by the authors.

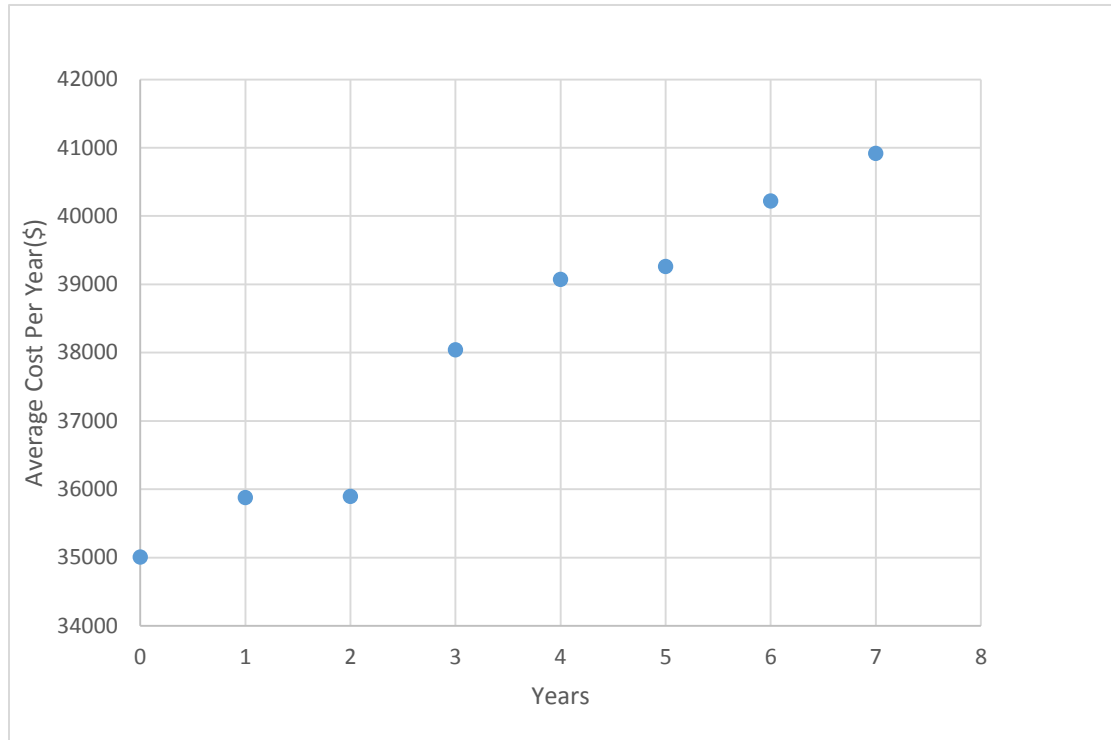For detailed background data and additional information, please visit http://trends.collegeboard.org.

4. After celebrating her 17th birthday, Jessica is planning to attend a private nonprofit four year college with room and board during the fall semester. Visit the following website and complete the table starting from the 2006-2007 academic year.

   http://trends.collegeboard.org/college-pricing/figures-tables/tuition-and-fee-and-room-and-board-charges-over-time

   (Students may refer to Page 13 to answer this question)

Academic Year	Year(t)	Average cost per year($) (*in 2013 Dollars*)
2006-2007	0	35007
2007-2008	1	*35878*
2008-2009	2	*35895*
2009-2010	3	*38041*
2010-2011	4	*39072*
2011-2012	5	*39261*
2012-2013	6	*40220*
2013-2014	7	*40917*

5. Draw the scatter plot for the above data by taking the time (x) in years on the x- axis and the average cost per year (in thousands) on the y-axis.

6. Which BEST describes the correlation of the two variables shown in the scatter plot above?

    A. weak positive
    B. strong positive
    C. weak negative
    D. strong negative

Part 3: Linear Regression Model

7. Use a technological tool to find the linear regression model for No. 4. Take the number of years (x), starting from academic year 2006- 2007, as an independent variable and the corresponding average cost per year as the dependent variable. Write your answer in the form f(x) = *a*x + *b*. Round the values of *a* and *b* to the nearest whole numbers. Interpret the values of **a** and **b** in the context of this problem.

> *Solution:*
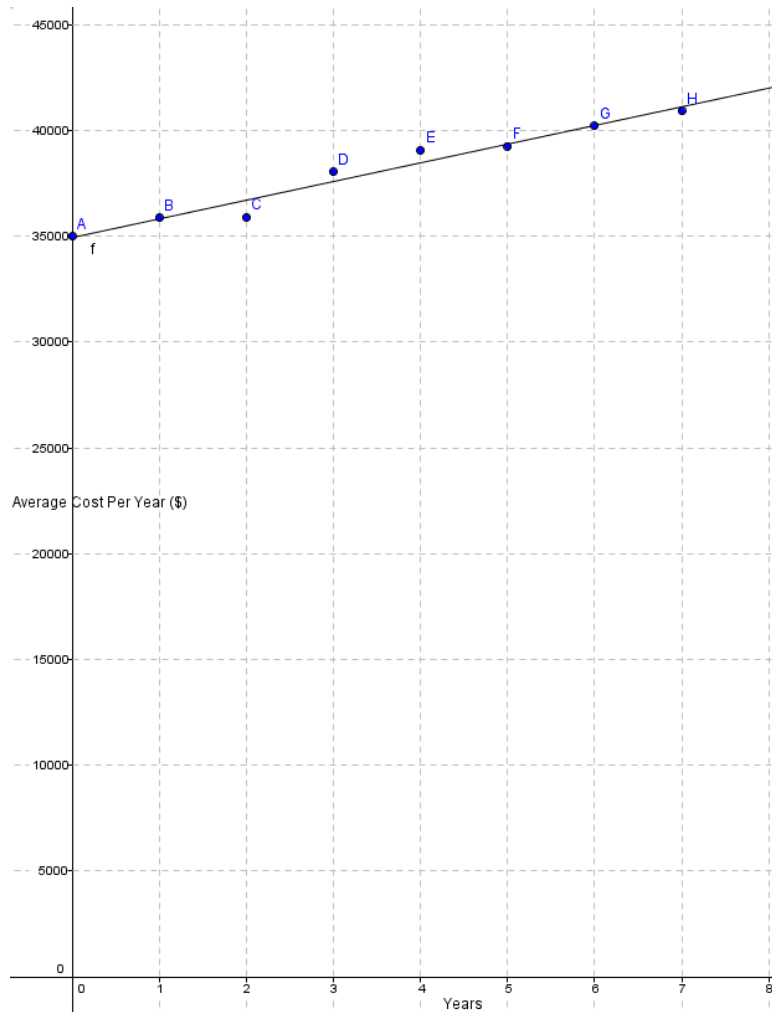> *f(x) = a x + b where a = 883 and b = 34944*
> *f(x) = 883 x + 34944*
>
> *Interpretation:*
> *The predicted average cost for the year 2006-2007 is $34944*
> *And the average annual increase in the cost is $ 883*

8. Draw the graph of the above linear regression model on the grid provided for No. 5.

9. In which academic year will Jessica be attending her first year of college?

   Solution:
   2024-2025 academic year

   Use the linear regression model from No.7 to answer questions 10-13. For No. 10-17, round your answer to the nearest dollar.

10. Predict Jessica's average cost for her first year's college education.

    Solution:
    Substitute x = 18 in f(x) = 883 x + 34944
    f(18) = 883(18) + 34944 = 50838
    Jessica's average predicted cost for her second year's college education = $50838

11. Predict Jessica's average cost for her second year.

Solution:
Substitute x = 19 in f(x) = 883 x + 34944
f(19) = 883(19) + 34944 = 51721
Jessica's average predicted cost for her third year's college education = $51721

12. Predict Jessica's average cost for her third year.

Solution:
Substitute x = 20 in f(x) = 883 x + 34944

f(20) = 883(20) + 34944 = 52604
Jessica's average predicted cost for her fourth year's college education = $52604

13. Predict Jessica's average cost for her fourth year.

Solution:
Substitute x = 21 in f(x) = 883 x + 34944

f(21) = 883(21) + 34944 = 53487
Jessica's average predicted cost for her fourth year's college education = $53487

14. What would be the total cost for all four of her college years?

Solution:
Total cost = $50838 + $51721 + $ 52604 + $53487= $208650

15. Jessica's institution is offering a 10% discount on the total fee if a student pays the fee for all four years in advance at the beginning of the first year. If she decides to pay the full amount in advance, what amount would she owe?

Solution:
90% of $208650 = $0.90 x 208650 = $187785

16. Does grandma's investment cover Jessica's total cost of her four year college education? Explain.

Sample solution:
No, Grandma's investment will cover only $97565

17. If not, how much more money should her grandma have invested on Jessica's first birthday?

Sample solution:

$187785 = P(1.0105)^{64}$

$P = \dfrac{187785}{(1.0105)^{64}} = 96236$

Grandma has to invest an additional amount $96236 - $50000 = $46236

Part 4: Exponential Regression Model

18. Use a technological tool to find the linear regression model for No. 4. Take the number of years (x), starting from academic year 2006- 2007, as an independent variable and the corresponding average cost per year as the dependent variable. Write your answer in the form $g(t) = a * b^x$. Round the value of **a** to the nearest whole number and **b** to the nearest six decimal places. Interpret the values **a** and **b** in the context of this problem.

> *Solution:*
>
> $g(x) = a * b^x$, *where a = 35003 and b = 1.023593*
>
> $g(x) = 35003 * (1.023593)^x$
>
> *Interpretation:*
>
> *The predicted average cost for the year 2006-2007 is $35003*
>
> *The average annual increase factor in the cost is 1.023593*

> Use the above exponential regression model from No.18 to answer questions19-22. For No. 19-26, round your answers to the nearest dollar.

19. Predict Jessica's average cost for her first year's college education.

> *Solution:*
>
> *Substitute x = 18 in* $g(x) = 35003 * (1.023593)^x$
>
> $g(18) = 35003 * (1.023593)^{18} = 53259$
>
> *Jessica's average predicted cost for her first year's college education = $53259*

20. Predict Jessica's average cost for her second year.

> *Solution:*
>
> *Substitute x = 19 in* $g(x) = 35003 * (1.023593)^x$
>
> $g(19) = 35003 * (1.023593)^{19} = 54516$
>
> *Jessica's average predicted cost for her first year's college education = $54516*

21. Predict Jessica's average cost for her third year.

Solution:
Substitute x = 20 in g(x) = $35003 * (1.023593)^x$
g(20) = $35003 * (1.023593)^{20}$ = 55802
Jessica's average predicted cost for her first year's college education = $55802

22. Predict Jessica's average cost for her fourth year of college.

Solution:
Substitute x = 21 in g(x) = $35003 * (1.023593)^x$
g(21) = $35003 * (1.023593)^{21}$ = 57119
Jessica's average predicted cost for her first year's college education = $57119

23. What would be the total cost for all of her four years in college?

Solution:
Total cost = $53259 + $54516 + $55802 + $57119= $220696

24. As you know, Jessica's institution is offering a 10% discount on the total fee if a student pays all four years fees at the beginning of the first year. Jessica and her grandma decided to opt for this discount program. How much Jessica will end up paying after discount?

Solution:
90% of $220696 = $0.90 x 220696 = $198626

25. Do you think, Grandma's investment will provide sufficient funds to pay Jessica's college cost/fee as you calculated in No.24? Justify.

Sample solution:
No, Grandma's investment will cover only $97565

26. If not, how much more money should her Grandma have invested on Jessica's first birthday?

Sample solution:

$$198626 = P(1.0105)^{64}$$

$$P = \frac{198626}{(1.0105)^{64}} = 101791$$

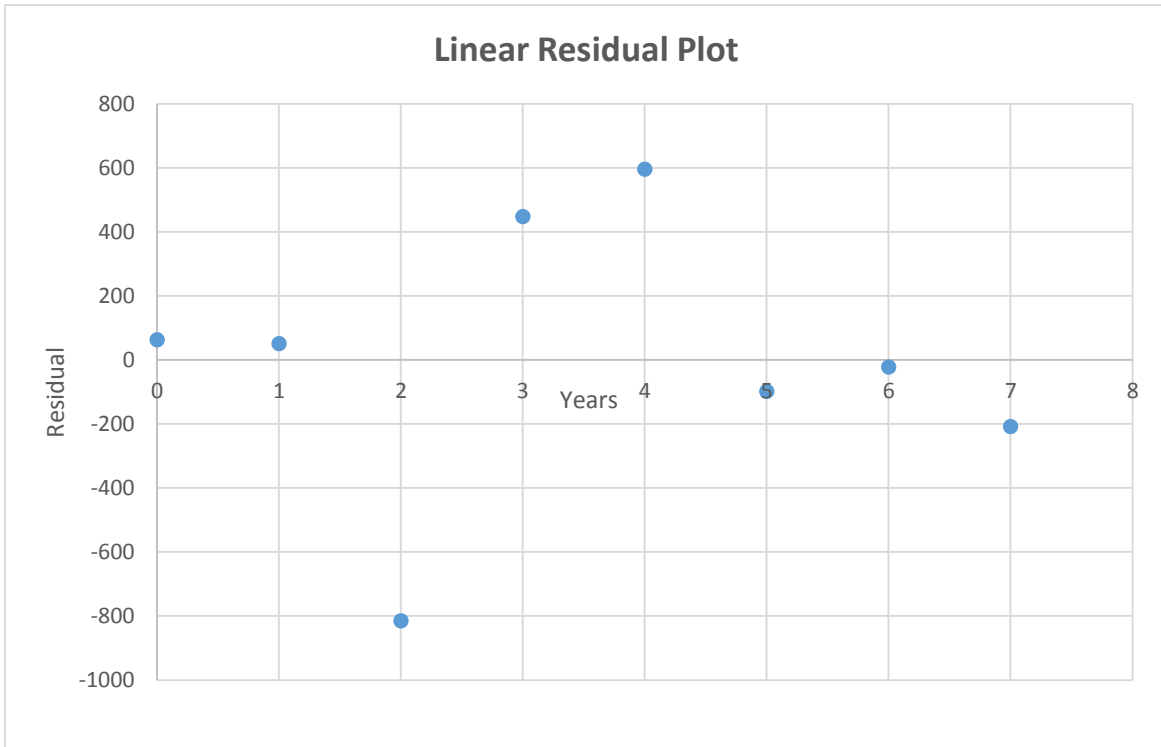*Grandma has to invest an additional amount $101791 - $50000 = $51791*

Part 5: Interpretation and Summarization

The goodness of fit depends on the model's accuracy in predicting values. **_Residuals,_** or error distances, are used to measure the goodness of fit. A residual is the difference between the observed value and the models predicted value. For a regression model, **a residual = observed value – predicted value**. A residual plot is a graph that shows the residual values (y) on the vertical axis and the independent variable (x) on the horizontal axis. A residual plot shows where the model fits best, and where the fit is worst. A good regression fit has very short residuals.

27. Calculate the residuals for the observed cost of education and the predicted cost of education by using the **linear regression model** you obtained from No. 7.

Academic Year	Year(x)	Average cost per year($) (Observed cost)	Predicted cost per year($) $f(x)=883x+34944$	Residual
2006-2007	0	35007	*34944*	*63*
2007-2008	1	*35878*	*35827*	*51*
2008-2009	2	*35895*	*36710*	*-815*
2009-2010	3	*38041*	*37593*	*448*
2010-2011	4	*39072*	*38476*	*596*
2011-2012	5	*39261*	*39359*	*-98*
2012-2013	6	*40220*	*40242*	*-22*
2013-2014	7	*40917*	*41125*	*-208*

28. Construct a residual plot for the above data by taking years on the x-axis and the corresponding residuals on the y-axis.

Linear Residual Plot

29. Determine if there are any patterns in the above residual plot and explain what they suggest about the relationship between the variables.

> *Solution:*
> *There is no pattern and a random scattering of points provides evidence to support a judgment that the model is appropriate for the data set.*
> *Explanation may vary.*

30. What is the sum of the residuals? Interpret the result.

> *Solution:*
> *The sum of the residuals is $15. The sum of the residuals is zero for the best regression model. The sum $ 15 is an insignificant value compare to the cost of education. Hence, the linear regression model is a good fit.*
> *Interpretation may vary.*

31. Do you think that the **linear regression model** you obtained is a good predictor of cost of education? Why or why not?

> *Solution:*
> *Answer may vary.*

32. Use a technology tool to calculate the correlation coefficient for the two variables, number of years (t) and the average cost of education. What can you conclude?

> *Solution:*
> *The correlation coefficient(r) = 0.98*
> *There is a 98% positive correlation between the academic years and the corresponding cost per year.*
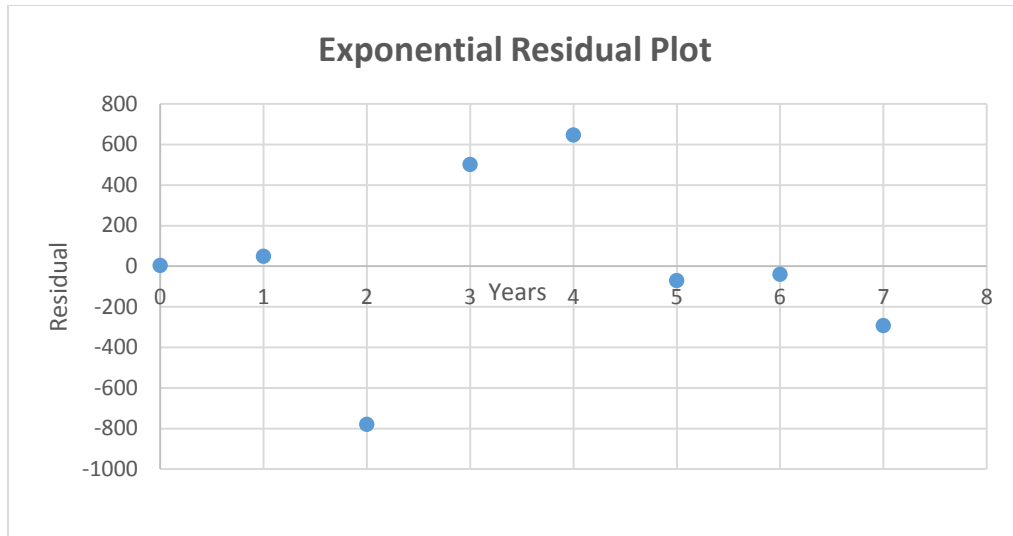
33. Calculate the residuals for the observed cost of education and the predicted cost of education by using the **exponential model** you obtained from question 18.

Academic Year	Year(x)	Average cost per year($) (Observed value)	Predicted cost per year($) $g(t)=35003(1.023593)^x$	Residual
2006-2007	0	35007	*35003*	*4*
2007-2008	1	*35878*	*35829*	*49*
2008-2009	2	*35895*	*36674*	*-779*
2009-2010	3	*38041*	*37539*	*502*
2010-2011	4	*39072*	*38425*	*647*
2011-2012	5	*39261*	*39332*	*-71*
2012-2013	6	*40220*	*40260*	*-40*
2013-2014	7	*40917*	*41209*	*-292*

34. Construct a residual plot for the above data obtained using the exponential model.

> *Solution:*
> *Observation may vary.*

Exponential Residual Plot

35. Determine if there are any patterns in the above residual plot and explain what they suggest about the relationship between the variables.

    Solution:
    There is no pattern and a random scattering of points provides evidence to support a judgment that the model is appropriate for the data set.

    Explanation may vary.

36. What is the sum of the residuals obtained using the exponential model? Interpret your answer.

    Solution:
    The sum of the residuals is $20. The sum of the residuals is zero for the best regression model. The sum $20 is an insignificant value compare to the cost of education. Hence, this exponential regression model is also a good fit.
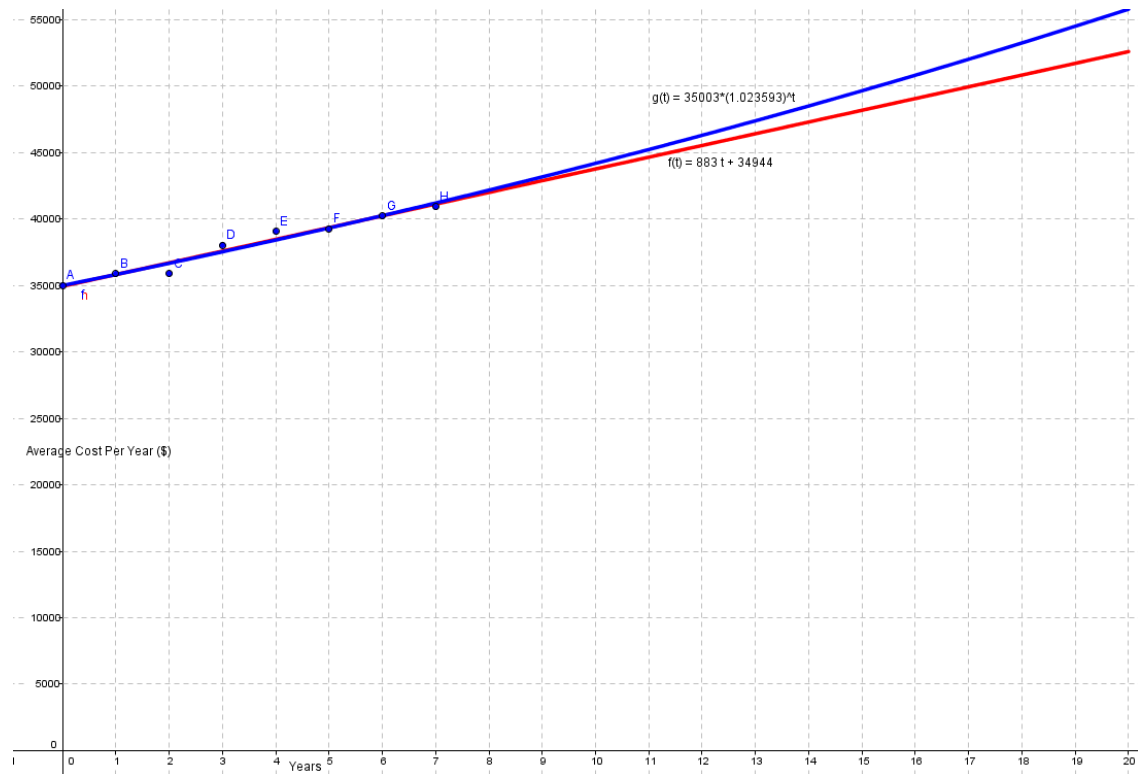   *Interpretation may vary.*

37. Do you think the **exponential regression model** you obtained is a good predictor of cost of education? Why or why not?

    Solution:
    Answer may vary.

38. Which one of these two models (**linear or exponential**) do you prefer? Justify your preference.

    Solution:

g(t) = 35003*(1.023593)^t

f(t) = 883 t + 34944

Answer may vary.

39. Using a graphing utility, draw the scatter plot for question No. 4, graph both linear and exponential models in a same grid. What is your observation?

       Solution:
       Observation may vary.

40. Write a paragraph about how this project will help you to plan for your college education.
       Solution:
       Answer may vary.

Trends in Higher Education

CollegeBoard

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Tuition and Fee and Room and Board Charges over Time

Other Data in this Topic ▾

Online Only!

The annual increase in inflation-adjusted average tuition and fees at public four-year colleges and universities has declined in each of the past five years, from 9.5% in 2009–10 to 0.9% in 2013-14.

| Academic Year | Tuition and Fees in 2013 Dollars | | | Tuition, Fees, Room and Board in 2013 Dollars | |
	Private Nonprofit Four-Year	Public Four-Year	Public Two-Year	Private Nonprofit Four-Year	Public Four-Year
71-72	$10,515	$2,456	$1,102	$16,811	$8,064
81-82	$10,489	$2,318	$1,107	$16,143	$7,319
91-92	$16,829	$3,614	$2,008	$24,334	$9,351
01-02	$22,869	$4,956	$2,116	$31,395	$11,886
06-07	$25,607	$6,662	$2,601	$35,007	$14,735
07-08	$26,264	$6,943	$2,573	$35,878	$15,204
08-09	$26,356	$7,008	$2,530	$35,895	$15,263
09-10	$27,920	$7,672	$2,787	$38,041	$16,526
10-11	$28,679	$8,174	$2,938	$39,072	$17,335
11-12	$28,830	$8,557	$3,074	$39,261	$17,739
12-13	$29,557	$8,816	$3,216	$40,220	$18,166
13-14	$30,094	$8,893	$3,264	$40,917	$18,391

| Academic Year | Tuition and Fees in Current Dollars | | | Tuition, Fees, Room and Board in Current Dollars | |
	Private Nonprofit Four-Year	Public Four-Year	Public Two-Year	Private Nonprofit Four-Year	Public Four-Year
71-72	$1,832	$428	$192	$2,929	$1,405
81-82	$4,113	$909	$434	$6,330	$2,870
91-92	$9,812	$2,107	$1,171	$14,188	$5,452
01-02	$17,377	$3,766	$1,608	$23,856	$9,032
06-07	$22,308	$5,804	$2,266	$30,497	$12,837
07-08	$23,420	$6,191	$2,294	$31,993	$13,558
08-09	$24,818	$6,599	$2,382	$33,800	$14,372
09-10	$25,739	$7,073	$2,569	$35,070	$15,235
10-11	$26,766	$7,629	$2,742	$36,465	$16,178
11-12	$27,883	$8,276	$2,973	$37,971	$17,156
12-13	$28,989	$8,646	$3,154	$39,447	$17,817
13-14	$30,094	$8,893	$3,264	$40,917	$18,391