

Sample Mathematics Learning Plan

Big Idea/ Topic

- Analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence

Standard(s) Alignment

Understand congruence and similarity using physical models, transparencies, or geometry software.

MGSE8.G.1 Verify experimentally the congruence properties of rotations, reflections, and translations: lines are taken to lines and line segments to line segments of the same length; angles are taken to angles of the same measure; parallel lines are taken to parallel lines.

MGSE8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

MGSE8.G.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.

MGSE8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.

MGSE8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.*

Diagnostic Assessment

Diagnostic Assessment Probe: Transformations

This assessment probe requires students to complete a series of transformations of a point, line segment and shape. Each student will need access to their own copy of the diagnostic.

If implementing synchronously or asynchronously:

- The slides can be uploaded into [Nearpod](#) or Desmos for students to provide input for each question. Students will need a platform in which annotation capabilities are available.

If implementing unplugged/offline:

- Provide each student with a hard copy of the diagnostic assessment.

Instructional Design

Engage (Problem-based Task)

Begin the conversation about transformations using the problem-based task, *How Did They Make Ms. Pac Man*, by Robert Kaplinsky. Play [The Situation video](#) for students. As students work with this task, it is okay for them to use imprecise language to describe precise thinking during this early learning phase.

- **Synchronous**

Pose the following question to students: How can you describe Ms. Pac-Man's movements? Allow students to have independent think-time after watching the video. Encourage students to record their thinking in preparation for a group share. After sufficient time, instruct students to work within small, collaborative groups to share their descriptions of Ms. Pac Man's movements. Consider using Google Slides, PowerPoint or [Padlet](#) to organize the groups. Ensure each group has a space to record their thinking. A whiteboard platform may be a useful feature to allow students to record the path they observed.

Look for groups who may have used descriptions similar to:

- left or right, up or down
- turn
- flipped or mirrored

Bring the class together for a whole class discussion. Strategically select groups to share their thinking. Consider using a discussion format such as this:

Layer 1- Identifying the movement: Call on the group who identified her movements as sliding right, left, up and down. Label the path with the direction she moved. Use this [video](#) to determine if the

slides were correctly identified. An alternative to using the video could be recreating the path movements under a document camera on a hard copy of a still image of the Ms. Pac Man clip.

Layer 2 - Identifying the turns: Select groups who identified any turns or rotations within Ms. Pac Man's movements. Replay the original video and students can shout "turn" each time she rounds a right angle or students can type "turn" in the chat window when she rounds a right angle.

Layer 3 - Briefly discuss what it would look like if she flipped instead of turned. Replay the original video and identify where she flipped or reflected.

Layer 4 - Summarizing: Help learners summarize the activity by creating an anchor chart. Google Slides or Padlet could be used to post vocabulary terms. Be sure to use strategies that honor students' thinking and provide accurate mathematical language.

(Possible) Student Language	Mathematical Terminology of Transformations
left or right, up or down	translation
turn	rotation
flip	reflection

- **Asynchronous**

Pose the following question to students: How can you describe Ms. Pac-Man's movements? Encourage students to record their thinking in preparation for a group share. After a determined amount of allotted time, instruct students to work within small collaborative groups to share their descriptions of Ms. Pac Man's movements using [Loom](#) (click [here](#) for a loom tutorial). Consider using Google Slides, PowerPoint or [Padlet](#) to organize the groups, ensure each group has a space to record their thinking. You may want to encourage students to record their name next to their thinking for accountability purposes. A whiteboard platform, such as [Jamboard](#) or [Google Draw](#) may be a useful feature to allow students to record the path they observed.

Look for groups who may have used descriptions similar to:

- left or right, up or down
- turn
- flipped or mirrored

Assign the [Part 1](#), [Part 2](#) and [Part 3](#) videos in consecutive order and provide students with a vocabulary chart similar to the one below:

(Possible) Student Language	Mathematical Terminology of Transformations
left or right, up or down	translation
turn	rotation
flip	reflection

Instruct students to identify in which video the transformations occurred.

- **Unplugged/ Offline**

Provide students with a copy [Ms. Pac-Man Challenges](#). In this activity, students are provided a screenshot of Mrs. Pac Man's starting and ending positions and must describe the transformations needed to get from beginning to end.

Pose the following question to students: How can you describe Ms. Pac-Man's movements? Encourage students to record their thinking and label the path with the direction she moved. Provide students with a vocabulary chart similar to the one below:

(Possible) Student Language	Mathematical Terminology of Transformations
left or right, up or down	translation
turn	rotation
flip	reflection

Explore

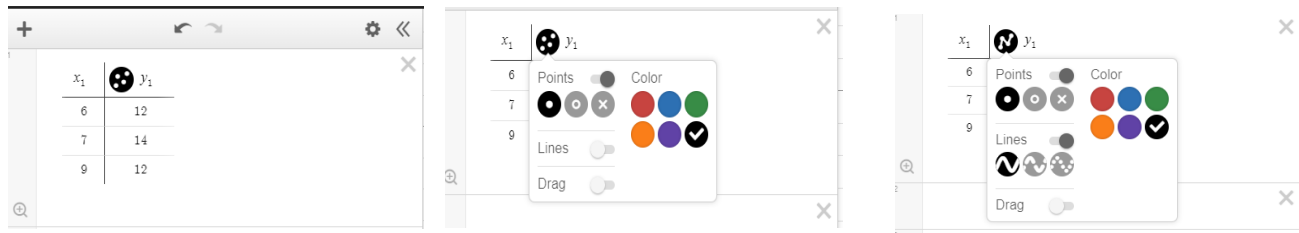
Students will engage in small group stations to explore 3 rigid transformations. As students begin the work, provide this essential question: Why are certain transformations considered rigid?

- **Synchronous/Asynchronous**

Station 1 (Translations)

1. Using the [Desmos Graphing Calculator](#), (instructions on how to use the calculator are located in the Appendix of the learning plan) have students plot the following three points to form a triangle: (6, 12), (7, 14), and (9, 12) by listing the points within the Desmos table. Use this duplicate point (6,

12), to create the enclosed figure. Have them connect the dots by clicking and holding down the icon next to the dependent column header. See image below for a visual of these directions.



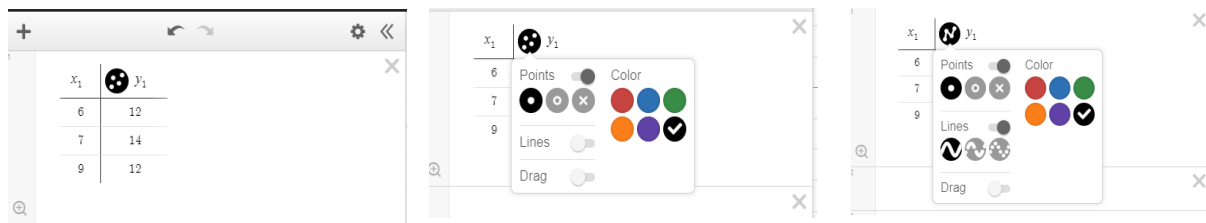
2. Have them create another table to plot the new vertices and connect in order to “slide” each vertex of the triangle down 2 spaces and right 1 space.

3. Repeat the procedure until six triangles have been plotted.

Station 2 (Reflections)

1. Using the [Desmos Graphing Calculator](#), (instructions on how to use the calculator are located in the Appendix of the learning plan) have students plot the following points: Note the duplicate point will result in a closed figure when using Desmos. (4, 4), (4, 4), (2, 10), (4, 12), and (6, 10). Students can connect the points by listing the points within the Desmos table. Have them connect the dots by clicking and holding down the icon next to the dependent column header. See image below for a visual of these directions.

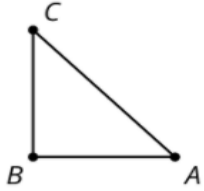
These represent the location of Mr. Lanka’s kite-shaped garden.



2. Have students reflect the image of the kite over the y-axis (the line of reflection) so that it is in the same position in Quadrant II as it is in Quadrant I. Have students use different color dots (using the same method for connecting the dots, but select a color) to plot this reflected image. Students should connect the points.

3. Have students compare the vertices of the pre-image to those of the image. Have students investigate the effect that reflecting a figure over the y-axis has on its coordinates.

Station 3 (Rotations)



Here is an isosceles right triangle: Draw these three rotations of triangle ABC together.

- A. Rotate triangle ABC 90 degrees clockwise around A .
- B. Rotate triangle ABC 180 degrees clockwise around A .
- C. Rotate triangle ABC 270 degrees clockwise around A .

• Unplugged/ Offline

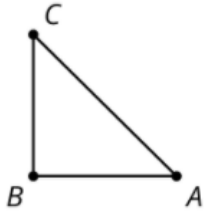
Station 1 (Translations)

1. Using the grid paper, have students plot the following three points to form a triangle: (6, 12), (6, 12), (7, 14), and (9, 12). Note: The duplicate point will result in a closed figure when using Desmos. Have them connect the dots.
2. Have students plot the new vertices and connect in order to “slide” each vertex of the triangle down 2 spaces and right 1 space.
3. Repeat the procedure until six triangles have been plotted.

Station 2 (Reflections)

1. Using the grid paper, have students plot the following points: (4, 4), (4, 4), (2, 10), (4, 12), and (6, 10). Note: The duplicate point will result in a closed figure when using Desmos. These represent the location of Mr. Lanka’s kite-shaped garden.
2. Have students reflect the image of the kite over the y -axis (the line of reflection) so that it is in the same position in Quadrant II as it is in Quadrant I. Have students use different color dots to plot this reflected image. Students should connect the points.
3. Have students compare the vertices of the pre-image to those of the image. Have students investigate the effect that reflecting a figure over the y -axis has on its coordinates.

Station 3 (Rotations)



Here is an isosceles right triangle: Draw these three rotations of triangle ABC together.

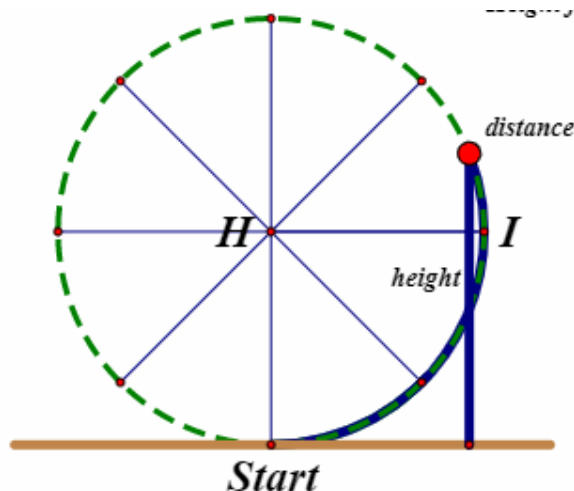
- A. Rotate triangle ABC 90 degrees clockwise around A .
- B. Rotate triangle ABC 180 degrees clockwise around A .
- C. Rotate triangle ABC 270 degrees clockwise around A .

Extension: Have students use one or two transformations to transform a figure. Display the student work to use as your example of congruent figures. Use the display to lead students to discover the definition of congruent.

Apply

You have been hired to assist a group of engineers at a local amusement park to design an exciting carnival ride. The amusement park owner wants you to design the world's greatest Ferris wheel for a local amusement park. Use a variety of tools to design and create the Ferris wheel and describe the transformations needed in order for the rider to have a thrilling and safe riding experience. Be sure to include details on the reflections and rotations involved in the Ferris wheel ride based on your design. Also, include a portion of the ride that includes a translation. Be sure to justify your thinking and outline all of the details of your design.

Consider beginning the conversation with this design of a traditional Ferris wheel. Encourage students to be creative in their designs; the images below may help spark design ideas.



Sample design ideas to share with students:



- **Synchronous/Asynchronous**

Students can use an online tool such as Desmos or AutoCAD to design their amusement park ride. Students can create a digital presentation to describe the details and show their design.

- **Unplugged/ Offline**

Students can design their amusement park ride on paper using graph paper and notebook paper to sketch their designs and describe the details.

Reflect

Students will engage in a formative assessment: Always, Sometimes, Never Activity

Pose the following questions and ask students to state if it is always true, sometimes true or never true.

1. Rotating a shape changes the features of the shape.
2. Reflections and rotations produce the same result.
3. If a triangle is rotated around the origin, all sides of the triangles will line up.
4. If a shape is translated down, the points of the image are negative.

- **Synchronous/Asynchronous**

Students can post their responses within a moderated discussion board or on a [Padlet](#) wall. Alternatively, the teacher can create a [Poll](#) using [Near Pod](#) and have the students answer the four statements

- **Unplugged/ Offline**

Provide each student with a copy of the reflection questions and space to justify their thinking. Consider using a structure similar to the one provided below in the Appendix.

Evidence of Student Success

Always, Sometimes or Never True activities involve a set of statements that students examine and decide if they are always true, sometimes true, or never true. This strategy is useful in revealing whether students overgeneralize or undergeneralize a mathematical concept. This formative assessment encourages teachers to ask students to examine the validity of statements and get into the habit of identifying examples that work and counterexamples that do not work.

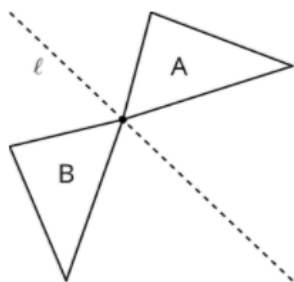
Technology Enhanced Assessment Task

[Geogebra](#) would be a good tool to enforce the definitions of congruence and similarity. The [linked activity, Congruent Figures: Dynamic Illustration](#), shows two congruent triangles and students have to slide and rotate one to see the congruence. This can be used to measure students' understanding.

Formative Assessment Task

Here is another assessment task that could be used to measure students' understanding of transformations.

- **Jada's Transformations**



Students will apply their understanding of rigid transformations to a new problem. Students can engage in a notice and wonder as they observe the characteristics of the two triangles. Pose the following question to students: ***Is Triangle B a rotation of Triangle A?***

Explain your reasoning either in your math journal, on notebook paper or using Google Docs or Microsoft Word to type your explanation and share with your teacher. You may also upload your typed explanations to your Learning Management System as a discussion post.

Jada applies two transformations to a polygon in the coordinate plane. One of the transformations is a translation and the other is a reflection. What information does Jada need to provide to communicate the transformations she has used?

Student Learning Supports

At all grades, the mathematics big ideas encourage students to reason mathematically, to evaluate mathematical arguments both formally and informally, to use the language of mathematics to communicate ideas and information precisely, and to make connections among mathematical topics and to other disciplines. The following strategies are intended to support students who are struggling to progress towards this goal:

- **Conceptual Processing:** Utilize the [Concrete-Representational-Abstract instructional sequence](#) to support students in making connections among mathematical ideas, facts and skills, and reflecting upon and refining one's own understanding of relationships, generalizations and connections.
- **Language:** Strategically select [language routines](#) to support students in describing strategies, explaining their reasoning, justifying solutions and making persuasive arguments.
- **Visual-Spatial Processing:** Provide opportunities for students to engage with visual representations and manipulatives (virtual or concrete) as they solve problems, explore concepts and communicate ideas.
- **Organization:** Teach problem-solving strategies and problem types, as seen in the [Mathematics Glossary: K – 12](#), in order to support students in figuring out how to get started, carrying out a meaningful sequence of steps while solving problems, keeping track of the information from prior steps, monitoring their own progress and adjusting strategies accordingly.
- **Memory:** Focus on conceptual strategies and patterns for computation, providing a scaffold for students who struggle with basic facts and carrying out written algorithms.

Here is a learning activity that can be used to support students' deepened understanding of the big idea.

- **[Congruent Rectangles](#).** In this task, students are given a series of transformations and must use these transformations to determine if the rectangles are congruent.
- **[Polygraph: Rigid Transformations](#).** This custom Polygraph is designed to spark vocabulary-rich conversations about rigid transformations. Key vocabulary that may appear in student questions includes: slide, shift, translation, spin, turn, rotation, flip, mirror or reflection.
- ***Differentiation Idea:*** *Some students may need to see an actual mirror to understand what reflections do, and the role of the reflection line. If you have access to rectangular plastic mirrors, you may want to have students check their work by placing the mirror along the proposed mirror line.*

Engaging Families

The [Open Up Resource Family Materials](#) resource from Illustrative Mathematics provides literature for parents/caregivers to understand the rationale of the strategies addressed within this unit. Sample problems are a part of this resource. Families are encouraged to work on the problems together and to engage their learner with these problems to strengthen his/her understanding of the big idea presented.

Appendix

Desmos Instructions

Tables

Dive deeper into data with tables! You can create a new table or convert an existing expression into a table. You can even copy/paste two column data from a spreadsheet into an empty expression box to quickly create a new table.



Add Table
Click the "Add Item" button to create a new table.

A screenshot of a Desmos table. The table has two columns labeled 'x' and 'y' and three rows. The data is as follows:

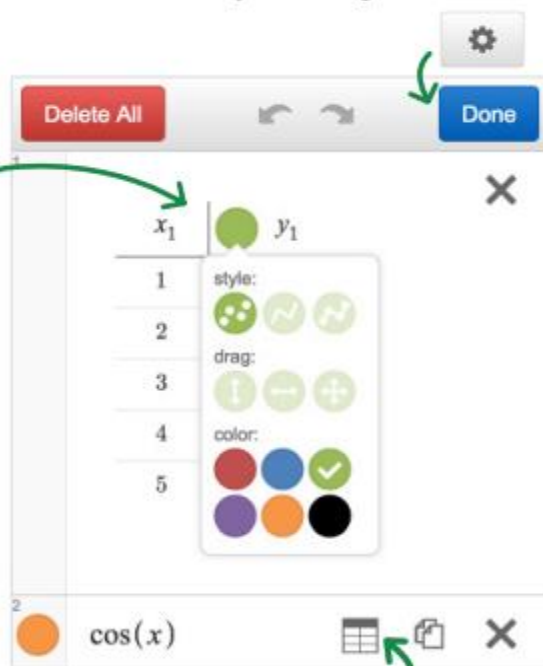
x	y
1	2
2	4
3	6

Green arrows point to the right arrow key and the down arrow key, indicating how to add a column or row respectively.

Add Column
Make a new column by hitting the right arrow key or clicking in the top cell of the column.

Add Row
Make a new row by hitting the down arrow key or clicking in the cell.

Options Menu
Click the icon to change color, connect or hide the points, or insert a new column.



In edit mode, click the "convert to table" icon to create a table from your expression.

Appendix
Reflection Assignment

Claim	Decision	Justification
1. Rotating a shape changes the features of the shape.	Always Sometimes Never	
2. Reflections and rotations produce the same result.	Always Sometimes Never	
3. If a triangle is rotated around the origin, all sides of the triangles will line up.	Always Sometimes Never	
4. If a shape is translated down, the points of the image are negative.	Always Sometimes Never	